Analytical model to calculate energy consumption

The energy consideration includes the following motion processes:

- Movement lift,
- movement shuttle carrier, and
- movement tote handling attachement.

The movements can be divided into two movement patterns:

- Movement with only acceleration and deceleration (case 1) and
- movement, with acceleration, deceleration and a constant velocity component (case 2).

Figure 1 shows the movement sequence for case 1.

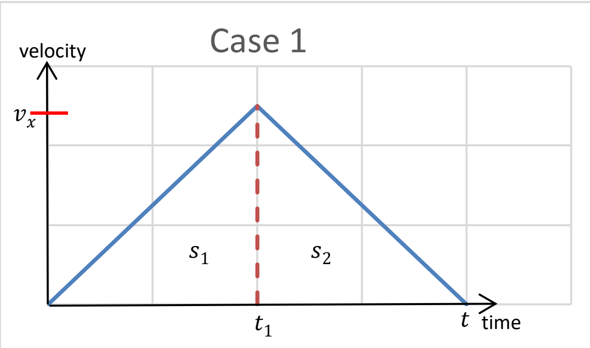

Figure 1: Movement sequence, case 1

s_1 is the distance traveled from the beginning to the end of the acceleration. s_2 is the distance traveled from the beginning to the end of the deceleration. The length of the distance s_1 has the numerical value of the area below the function line to t_1. t_1 is the time to reach v_x. t is the time required for the movement, from the beginning of the movement to the standstill of the mass. If acceleration and deceleration are identical, then s_1 and s_2 are equal ($s_1 = v_x * \frac{t_1}{2} = s_2$). For different acceleration and deceleration values is $s_2 = v_x * \frac{t_2 - t_1}{2}$. v_x is the achieved velocity. v_x is less than or equal to the maximum achievable velocity v.

Figure 3 shows the movement sequence for case 2.

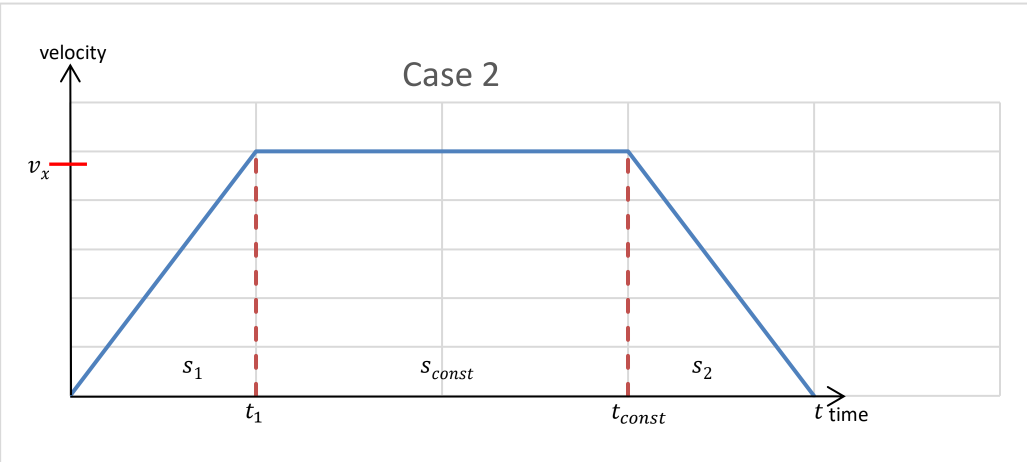

Figure 2: Case 2

s_{const} is the distance traveled at constant velocity.

Calculation of the critical distance to make a case distinction

Case 1 occurs when the distance to be traveled is less than or equal to the distance at which the maximum velocity is reached. Case 2 occurs when the distance to be traveled is greater than the distance at which the maximum velocity is reached. The distance required to reach the maximum velocity (and then immediately start with the deceleration) is referred to as the critical distance s_{crit}.

The critical distance is calculated as follows:

$$s_{crit} = \frac{a_1}{2}(t_1)^2 + \frac{a_2}{2}(t - t_1)^2$$
$$s_{crit} = \frac{a_1}{2}\left(\frac{v}{a_1}\right)^2 + \frac{a_2}{2}\left(\frac{v}{a_2}\right)^2$$
$$s_{crit} = \frac{1}{2}v^2\left(\frac{1}{a_1} + \frac{1}{a_2}\right) \quad (1)$$

a_1 denotes the acceleration, a_2 the deceleration.

Calculation of the travel time for case 1

It applies $s \leq s_{crit}$.

The travel time for case 1 is calculated as follows:

$$s = \frac{1}{2}{v_x}^2\left(\frac{1}{a_1} + \frac{1}{a_2}\right)$$
$$v_x = \sqrt{\frac{2s}{\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}}$$
$$t = \frac{v_x}{a_1} + \frac{v_x}{a_2}$$
$$t = \left(\frac{1}{a_1} + \frac{1}{a_2}\right)\sqrt{\frac{2s}{\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}} \quad (2)$$

Calculation of the travel time for case 2

It applies $s > s_{crit}$.

The travel time for case 2 is calculated as follows:

$$t = \frac{v}{a_1} + \frac{v}{a_2} + \frac{s - s_{crit}}{v}$$
$$t = \frac{v}{a_1} + \frac{v}{a_2} + \frac{s - \frac{1}{2}v^2\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}{v}$$
$$t = \frac{v}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right) + \frac{s}{v} \quad (3)$$

Calculation of the energy consumption of the vertical movement (elevator)

For the calculation of the energy consumption of the elevator movement, it is assumed that the elevator needs energy when moving upwards, and that it can maybe recover energy while moving downwards. If no energy can be recovered, since no recuperation is used, the recuperation efficiency is 0 percent, so no energy is consumed and none is recovered for moving downwards.

The energy consumption of an elevator movement upwards is calculated as follows:

$$E_{Elevator,up} = \frac{1}{\eta_{Elevator}}(m_{Elevator} + m_{Shuttle} + m_{Tote}) * g * s_{Elevator} \quad (4)$$

$\eta_{Elevator}$ is the efficiency factor of the elevator. $m_{Elevator}$ is the moving mass of the elevator. $m_{Shuttle}$ is the mass of the shuttle carrier. For tier-captive SBS/RS applies $m_{Shuttle} = 0$, since no shuttle carriers are transported by the elevator. m_{Tote} is the mass of the tote. If no tote is transported by the elevator, then $m_{Tote} = 0$. g is the gravitational acceleration. $s_{Elevator}$ is the vertical distance traveled.

The energy demand of an elevator movement downwards is calculated as follows:

$$E_{Elevator,down} = -\eta_{Elevator,Rekup} * (m_{Elevator} + m_{Shuttle} + m_{Tote}) * g * s_{Elevator} \quad (5)$$

$-\eta_{Elevator,Rekup}$ is the recuperation efficiency factor of the elevator. If no recuperation is used, $-\eta_{Elevator,Rekup} = 0$. A negative value for the energy consumption corresponds to an energy recovery.

Calculation of the energy requirement of the horizontal movement (shuttle carrier or tote handling attachement) for case 1

Figure 3 shows the balance of forces of the horizontal movement during the acceleration process. The air resistance is neglected, due to low estimated impact.

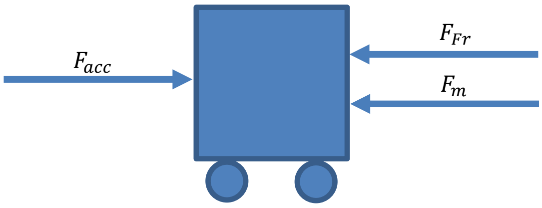

Figure 3: Balance of forces horizontal mass movement during acceleration

F_{acc} is the force needed to accelerate the mass. F_{Fr} is the friction force. F_m is the force of mass inertia.

The friction force is calculated as follows:

$$F_{Fr} = \mu_r * F_N = \mu_r * F_G = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g$$

m_H denotes the mass of the tote handling attachment (only the moving mass of it). μ_r denotes the coefficient of friction. When calculating a movement with a shuttle carrier, μ_r uses the coefficient of friction for the shuttle carrier, and when there is movement of the tote handling attachment, the coefficient of friction is applied to the tote handling attachment. When a movement of the tote handling attachment is calculated, then $m_{Shuttle} = 0$. If there is no tote on the shuttle carrier unit during the move, then $m_{Tote} = 0$.

The force of mass inertia is calculated as follows:

$$F_m = (m_{Shuttle} + m_{Tote} + m_H) * a_1$$

The balance of forces of horizontal acceleration gives:

$$F_{acc} = F_m + F_{Fr} = (m_{Shuttle} + m_{Tote} + m_H) * a_1 + \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g$$

The energy requirement for horizontal acceleration thus results:

$$E_{acc} = F_{acc} * s_1$$

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s_1 + (m_{Shuttle} + m_{Tote} + m_H) * a_1 * s_1$$

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s_1 + (m_{Shuttle} + m_{Tote} + m_H) * a_1 * \frac{1}{2} a_1 {t_1}^2$$

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s_1 + (m_{Shuttle} + m_{Tote} + m_H) * a_1 * \frac{1}{2} a_1 \frac{{v_x}^2}{{a_1}^2}$$

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * \mathrm{g} * \mathrm{s}_1 + \frac{1}{2} (m_{Shuttle} + m_{Tote} + m_H) {v_x}^2$$

Ancillary calculation to express s_1 as a function of acceleration and deceleration and total distance traveled:

$$v_x = \sqrt{\frac{2s}{\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}}$$

$$s_1 = \frac{1}{2} a_1 {t_1}^2$$

$$t_1 = \frac{v_x}{a_1}$$

$$s_1 = \frac{1}{2} a_1 \frac{{v_x}^2}{{a_1}^2}$$

$${v_x}^2 = \frac{2s}{\left(\frac{1}{a_1} + \frac{1}{a_2}\right)} = 2s * \frac{a_1 * a_2}{a_1 + a_2}$$

$$s_1 = \frac{1}{2} \frac{\left(\sqrt{\frac{2s}{\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}}\right)^2}{a_1}$$

$$s_1 = \frac{1}{2}\frac{\left(\frac{2s}{\left(\frac{1}{a_1}+\frac{1}{a_2}\right)}\right)}{a_1}$$

$$s_1 = s\frac{\left(\frac{1}{\left(\frac{1}{a_1}+\frac{1}{a_2}\right)}\right)}{a_1}$$

$$s_1 = \frac{s}{1+\frac{a_1}{a_2}}$$

$$s_1 = s\frac{a_2}{a_2+a_1}$$

Thus, the energy consumption of the acceleration can be formulated as follows:

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s\frac{a_2}{a_1+a_2} + (m_{Shuttle} + m_{Tote} + m_H) * s * \frac{a_1 * a_2}{(a_1+a_2)}$$

If we substitute $a = \frac{a_1 * a_2}{(a_1+a_2)}$, then follows:

$$E_{acc} = (m_{Shuttle} + m_{Tote} + m_H) * s * a * \left(\mu_r * g * \frac{1}{a_1} + 1\right) \quad (6)$$

With consideration of an efficiency factor η_h, the energy requirement is:

$$E_{acc,h} = \frac{1}{\eta_h}(m_{Shuttle} + m_{Tote} + m_H) * s * a * \left(\mu_r * g * \frac{1}{a_1} + 1\right) \quad (7)$$

When a movement is calculated for a shuttle carrier, the efficiency factor for the shuttle carrier is used, when a movement of the tote handling attachement is calculated, the efficiency factor of the tote handling attachement is used.

In the following, the energy recovery during the deceleration is calculated. Figure 4 shows the balance of forces of horizontal mass movement during braking. Air resistance is neglected.

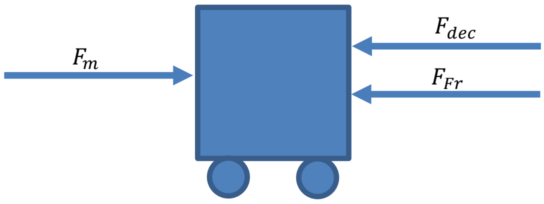

Figure 4: Force balance horizontal mass movement during deceleration

The balance of forces of the deceleration gives:

$$F_{dec} = F_m - F_{Fr}$$

The energy recovery of the deceleration thus results in:

$$E_{dec} = (F_m - F_{Fr})s_2$$

$$E_{dec} = (m_{Shuttle} + m_{Tote} + m_H) * s * a - \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s\frac{a_1}{a_1+a_2}$$

$$E_{dec} = (m_{Shuttle} + m_{Tote} + m_H) * s * a * (1 - \mu_r * g * \frac{1}{a_2}) \quad (8)$$

With consideration of the efficiency of the recuperation $\eta_{h,r}$, the energy recovery results in:

$$E_{dec,h} = \eta_{h,r}(m_{Shuttle} + m_{Tote} + m_H) * s * a * (1 - \mu_r * g * \frac{1}{a_2}) \quad (9)$$

The total energy consumption for case 1 is calculated as follows:

$$E_{case1,h} = E_{acc,h} - E_{dec,h}$$

$$E_{case1,h} = \frac{1}{\eta_h}(m_{Shuttle} + m_{Tote} + m_H) * s * a * \left(\mu_r * g * \frac{1}{a_1} + 1\right) -$$

$$-\eta_{h,r}(m_{Shuttle} + m_{Tote} + m_H) * s * a * (1 - \mu_r * g * \frac{1}{a_2})$$

$$E_{case1,h} = (m_{Shuttle} + m_{Tote} + m_H) s * a \left(\frac{1}{\eta_h}\left(\mu_r * g * \frac{1}{a_1} + 1\right) - \eta_{h,r}\left(1 - \mu_r * g * \frac{1}{a_2}\right)\right) \quad (10)$$

Calculation of the energy consumption of the horizontal movement (shuttle carrier or tote handling attachement) for case 2

In the case of the accelerated movement, in case 2, the achieved velocity is known (maximum velocity), therefore, the formulas for calculating the acceleration and the deceleration can be simplified as follows.

The energy consumption for the acceleration is calculated as follows:

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s_1 + \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2$$

$$E_{acc} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * \frac{v^2}{2a_1} + \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2$$

$$E_{acc} = \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2 * \left(\mu_r * g * \frac{1}{a_1} + 1\right) \quad (11)$$

With consideration of the efficiency factor η_h, the energy consumption of the acceleration results:

$$E_{acc,h} = \frac{1}{2\eta_h}(m_{Shuttle} + m_{Tote} + m_H) v^2 * \left(\mu_r * g * \frac{1}{a_1} + 1\right) \quad (12)$$

The energy recovery of the deceleration is calculated as follows:

$$E_{dec} = \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2 * (1 - \mu_r * g * \frac{1}{a_2}) \quad (13)$$

Including the efficiency factor of the energy recovery $\eta_{h,r}$ of the deceleration results in:

$$E_{dec,h} = \frac{\eta_{h,r}}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2 * (1 - \mu_r * g * \frac{1}{a_2}) \quad (14)$$

The energy consumption of the constant velocity movement is calculated below. The balance of forces is shown in Figure 5.

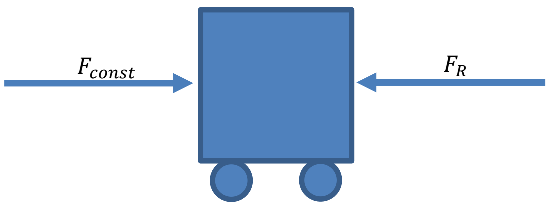

Figure 5: Balance of forces horizontal mass movement during constant velocity motion

The balance of forces results:

$$F_{const} = F_R$$

The energy consumption of the movement with constant velocity results in:

$$E_{const} = F_R * s_{const}$$

$$E_{const} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s_{const}$$

$$E_{const} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * (s - s_1 - s_2)$$

$$E_{const} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * \left(s - \frac{v^2}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)\right) \quad (15)$$

Taking into account the efficiency factor η_h results in the energy consumption:

$$E_{const,h} = \frac{1}{2\eta_h}\mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * \left(s - \frac{v^2}{2}\left(\frac{1}{a_1} - \frac{1}{a_2}\right)\right) \quad (16)$$

The following summarizes the energy consumption of accelerated motion and constant motion:

$$E_{acc} + E_{const} = \mu_{roll} * (m_{Shuttle} + m_{Tote} + m_H) * g * s_1 + \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2 + \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * s_{const}$$

$$E_{acc} + E_{const} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * (s_1 + s_{const}) + \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2$$

$$E_{acc} + E_{const} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * (s - s_2) + \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H) v^2$$

$$E_{acc} + E_{const} = \mu_r * (m_{Shuttle} + m_{Tote} + m_H) * g * (s - \frac{v^2}{2a_2}) + \frac{1}{2}(m_{Shuttle} + m_{Tote} + m_H)v^2$$

$$E_{acc} + E_{const} = (m_{Shuttle} + m_{Tote} + m_H)\left(\mu_r * g * \left(s - \frac{v^2}{2a_2}\right) + \frac{1}{2}v^2\right) \quad (17)$$

With consideration of the efficiency factor η_h, the energy consumption is:

$$E_{acc,h} + E_{const,h} = \frac{1}{2\eta_h}(m_{Shuttle} + m_{Tote} + m_H)\left(\mu_r * g * \left(s - \frac{v^2}{2a_2}\right) + \frac{1}{2}v^2\right) \quad (18)$$

The total energy consumption for case 2 is:

$$E_{case2,h} = E_{acc,h} + E_{const,h} - E_{dec,h}$$

$$E_{case2,h} = \frac{1}{2\eta_h}(m_{Shuttle} + m_{Tote} + m_H)\left(\mu_r * g * \left(s - \frac{v^2}{2a_2}\right) + \frac{1}{2}v^2\right) - \frac{\eta_{h,r}}{2}(m_{Shuttle} + m_{Tote} + m_H)v^2 * \left(1 - \mu_r * g * \frac{1}{a_2}\right)$$

$$E_{case2,h} = (m_{Shuttle} + m_{Tote} + m_H)\left(\frac{1}{2\eta_h}\left(\mu_r * g * \left(s - \frac{v^2}{2a_2}\right) + \frac{1}{2}v^2\right) - \frac{\eta_{h,r}}{2}v^2 * \left(1 - \mu_r * g * \frac{1}{a_2}\right)\right) \quad (19)$$